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Upshot On Altering Distances between The Points with Accretion on Fixed Point Theorems

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ABSTRACT

In contemporary discussion article , we have confirmed two results. The first is an accretion of Sayyed et.al. [10] and Sayyed and Sayyed [12] and the next is a pair of self-mappings of a metric space by altering distances between points with the use of certain control functions .

Keywords and phrases: Common Fixed Point, Fixed point, Metric space, Selfmaps, Orbitally continuous.

AMS (2000) Subject Classifications: Primary 54H25; Secondary 47H10.

1. Introduction And Preliminaries

In this manner several authors extended, expanded, established, obtained, generated fixed point results in metric space, selpmaps, control function altering distances , here we are giving some names like Khan et. al. [6], Sastry and Babu [8], Gairola and Krishan [2], Sastry. et. al. [9],Hosseni and Hosseni[3], Jain anddd Sayye [4.5]Naidu [7],Singh and Dimri [13],Babu and Ismail [1],Sayyed et.al. [10,12], Sayyed and Sayyed [11], Mane and Sayyed [5]proved some results on fixed point theorems for selfmaps. In our work we are referring the same concept and definition of Sayyed et.al. [10,12] and Sayyed and Sayyed [11]without changing with theorem proved by Khan, Swaleh and Sessa [6]and Sastry and Babu [8]. Here we gave a result which is accretion of sayyed et.al[12].

Theorem 1.1. Let M be a metric space, assume that a point $m_0 \in$ M such that the orbit $O(m_0) = \{S^n m_0 : n = 0, 1,2, \ldots..\}$ has a cluster point $z \in M$. If S is orbitally continuous at z and there exists a $\phi \in \Phi$ such that

$$\emptyset\big(d(Sx,Sy)\big) \le \frac{\alpha[\emptyset(d(x,Sx))\emptyset(d(y,Sy))]}{\emptyset(d(x,y))} + \beta\{\emptyset\,(d(x,Sx) + \emptyset\,(d(y,Sy)\} + \gamma\,\emptyset\,(d(x,y)) \quad \text{--- }(*)$$

Where α, β and γ are non-negative with $\alpha+2\beta+\gamma < 1$ for each $x,y = Sx \in O(m_0)$, $x\neq y$, then z is the unique fixed point of S

Proof . Define the sequence $\{m_n\}$ by $m_n = S^n m_0$ for n = 1,2,… . If $m_n = m_{n+1}$ for some n, then $m_m = m_n$ for $m \geq n$ so that $z = \lim_{m\to\infty} m$ and Sz = z by the orbital continuity of S.

We now assume $m_n \neq m_{n+1}$ for every n in N. If $\alpha_n = \phi[d(m_n,m_{n+1})]$, then by (*) it follows that

$\alpha_{n+1} = \phi\,[d(m_{n+1},m_{n+2})] = \phi\,[d(Sm_n,Sm_{n+1})]$

$$\leq \alpha \frac{\emptyset(d(m_n,Sm_n))\emptyset(d(m_{n+1},Sm_{n+1}))}{\emptyset(d(m_n,m_{n+1}))} + \beta\left\{\emptyset\left(d(m_n,Sm_n)\right) + \emptyset\left(d(m_{n+1},Sm_{n+1})\right)\right\} + \gamma\,\emptyset\,(d(m_n,m_{n+1}))$$

$$= \frac{\alpha\,\emptyset\left(d(m_n,m_{n+1})\right)\emptyset(d(m_{n+1},m_{n+2}))}{\emptyset(d(m_n,m_{n+1}))} + \beta\{\emptyset(d(m_n,m_{n+1})) + \emptyset(d(m_{n+1},m_{n+2}))\} + \gamma\,\emptyset(d(m_n,m_{n+1}))$$

$$= \alpha\,\alpha_{n+1} + \beta\alpha_n + \beta\alpha_{n+1} + \gamma\,\alpha_n$$

$\frac{\alpha_n+1}{\alpha_n} \leq k$ $\forall$ n ∈ N. Where $k = \frac{\beta+\gamma}{1-\alpha-\beta}$.

Hence $\alpha_{n+1} \leq k\alpha_n$ $\forall$ n ∈ N.

So that $\{\alpha_n\}$ is a strictly decreasing sequence of positive numbers and hence converges, say to $j \geq 0$ and $j \leq j\gamma$. Hence j = 0.

Let k(n) be a sequence of positive numbers such that $\{m_{k(n)}\}$ converges to z. Then $\{\alpha_{k(n)}\}$ converges to γ. By the continuity of ϕ and orbital continuity of S at z.

$J = \phi\,[d(m_{k(n)}, m_{k(n)+1})] = \phi\,[d(z, Sz)]$.

Since j = 0 and ϕ(t) = 0 => t = 0, it follows that Sz = z.

To prove uniqueness, let u(≠z) be another fixed point of S. Then

$\phi\,(d(z,u)) = \phi\,(d(Sz, (Su)))$

$$\leq \frac{\alpha\,\emptyset(d(z,S_z))\emptyset(d(u,S_u))}{\emptyset(d(z,u))} + \beta\{\emptyset(d(z,S_z)) + \emptyset(d(u,S_u))\} + \gamma\emptyset(d(z,u)) \qquad \text{(by *)}$$

$= (\beta+\gamma)\,\phi\,(d\,(z, u))$

i.e, $\phi\,(d\,(z, u)) \leq (\beta+\gamma)\phi(d\,(z, u))$

i.e. $(1-\beta-\gamma)\,\phi\,(d\,(z, u)) \leq 0$

which is a contradiction and hence z is the only fixed point of S. This completes the proof.

2. Fixed Point Theorems For A Pair of Selfmaps

Theorem 2.1. Let G* and H* be selfmaps of a metric space M. For $m_0 \in M$, define the sequence $\{m_n\}$ by $m_{2n+1} = G^*m_{2n}$, $m_{2n+2.} = H^*m_{2n+1.}$ Suppose either $\{m_{2n}\}$ has a cluster point z in M; and

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G^*, H^*G^* are continuous at z (or) $\{m_{2n+1}\}$ has a cluster point z in M; and H^*,G^*H^* are continuous at z. Assume that there exists a $\phi \in \Phi$ such that

$$\phi [d(G^*x,H^*y)] < \frac{1}{4} \{\phi\, d(x,G^*x) + \phi d(y,H^*y)+ \phi d(x,H^*y) +\phi d(y,G^*x) \} \qquad \text{--(3.1)}$$

for each distinct $x,y \in \{m_n\}$ satisfying either $x = H^*y$ or $y = G^*x$. Then either G^* or H^* has a fixed point in $\{m_n\}$ or z is a common fixed point of G^* and H^*

Proof. Suppose $m_{2n} = m_{2n+1}$, for some n, then $m_{2n} = m_{2n+1} = G^*m_{2n}$ so that m_{2n} is a fixed point of G^*. If $m_{2n+1} = m_{2n+2}$ for some n, then $m_{2n+1} = H^*m_{2n+1}$ so that m_{2n+1} is a fixed point of H^*. Suppose $m_n \neq m_{n+1}$ for all n. We write

$\alpha_n = \phi [d(m_n, m_{2n+1})]$ then

$$\begin{aligned}
\alpha_{2n} &= \phi [d(m_{2n},m_{2n+1})] = \phi [d(H^*m_{2n-1},G^*m_{2n})] \\
&< \frac{1}{6}\{\phi(m_{2n-1},H^*m_{2n-1}) +\phi d(m_{2n},G^*m_{2n}) \\
&\qquad +\phi d(m_{2n-1},G^*m_{2n})+\phi d(m_{2n},H^*m_{2n-1})\} \\
&= \frac{1}{6} \{\phi\, d(m_{2n-1},m_{2n}) + \phi\, d(m_{2n},m_{2n+1}) \\
&\qquad + \phi\, d(m_{2n-1},m_{2n+1}) + \phi\, d(m_{2n},m_{2n-1})\} \\
&< \frac{1}{6} [3\alpha_{2n-1} +2\alpha_{2n})]
\end{aligned}$$

$$\alpha_{2n} < \alpha_{2n-1}$$

Similarly $\alpha_{2n+1} < \alpha_{2n}$ holds.

Hence $\{\alpha_n\}$ is a strictly decreasing sequence of positive terms converges to a limit, say α.

Suppose either part holds. Then there exists a sequence $\{k(n)\}$ of positive integers such that $\{M_{2k(n)}\}$ converges to z. Then $\{\alpha_{2k(n)}\}$ converges to α. Now, by the continuity of S at z.

$$\begin{aligned}
\alpha = \lim_{n\to\infty} \alpha_{2k(n)} &= \lim_{n\to\infty} \emptyset[d(m_{2k(n)}, m_{2k(n)+1})] \\
&= \lim_{n\to\infty} \emptyset[d(m_{2k(n)}, G * m_{2k(n)})] \\
&= \phi [d(z,G^*z)] = \alpha
\end{aligned}$$

If $\alpha = 0$ then $G^*z = z$.

If $\alpha \neq 0$ then by continuity of G^* and H^*G^* at z, we have

$$\begin{aligned}
\alpha = \lim_{n\to\infty} \alpha_{2k(n)+1} &= \lim_{n\to\infty} \emptyset[d(m_{2k(n)+1}, m_{2k(n)+2})] \\
&= \lim_{n\to\infty} \emptyset[d(G * m_{2k(n)}, (H * G *)m_{2k(n)})] \\
&= \phi [d(z,G^*z)] = \alpha
\end{aligned}$$

so that $\alpha = \phi [d(G^*z,H^*G^*z)] < \phi [d(z,G^*z)] = \alpha$

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a contradiction. Hence α must be zero. Therefore G*z = z. Then z is a common fixed point of G* and H*.

Conclusion

The upshot of altering distances between points through the accretion framework significantly strengthens the scope and applicability of fixed-point theorems. By replacing classical metric contractions with altering distance functions, the restrictive nature of traditional Lipschitz-type conditions is relaxed, allowing a wider class of nonlinear mappings to be analyzed. Accretive conditions on fixed points ensure stability and convergence while preserving the uniqueness and existence results fundamental to fixed point theory. This generalized approach not only unifies several existing contraction principles but also extends them to more abstract metric and normed spaces. Consequently, fixed point results obtained under altering distance conditions demonstrate greater flexibility in handling complex functional equations.

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