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# Dyon Condensation and Dual Superconductivity in Quantum Chromodynamics: An Abelian Higgs Model Approach

**Ritu**
CSIR NET / JRF Qualified in physical Science

**Abstract**
This study investigates dyon condensation and the emergence of dual superconductivity within the Abelian Higgs framework of Quantum Chromodynamics (QCD). By analyzing these non-perturbative mechanisms, the work offers deeper insight into the complex behavior of QCD under extreme conditions, such as high temperature and high baryon density. Dyons particles carrying both electric and magnetic charges exhibit a distinctive response in which they screen the gauge potentials to which they are directly coupled, while simultaneously inducing antiscreening effects in the corresponding dual potentials. This interplay is consistent with a generalized Meissner effect and leads naturally to the realization of dual superconductivity. The effective action for dyonic fields, obtained through the Abelian projection of QCD, explicitly demonstrates this mechanism and reinforces the confinement scenario via the behavior of the Wilson loop within the Abelian Higgs model. This formulation provides a coherent theoretical description of confinement and phase transitions in strongly interacting matter. The results presented here deepen our understanding of the QCD vacuum structure and have important implications for both theoretical and experimental physics, with relevance extending from particle physics to cosmology. Ultimately, this study contributes to a more unified and comprehensive picture of the fundamental interactions governing matter in the universe.

**Keywords:** *Dyon, Condensation, Dual Superconductivity, Abelian, Higgs Model, Quantum Chromodynamics (QCD)*

## 1.INTRODUCTION

The investigation of dyon condensation and dual superconductivity within the Abelian Higgs framework of Quantum Chromodynamics (QCD) represents an important and active area of research in theoretical physics. QCD, the fundamental theory of the strong interaction, governs the dynamics of quarks the constituents of protons, neutrons and other hadrons and their interactions mediated by gluons. Although QCD has achieved remarkable success in describing high-energy particle processes, developing a comprehensive understanding of quark and gluon behavior under extreme conditions, such as those present in the early universe or in the dense interiors of neutron stars, remains a formidable challenge. To address these complexities, effective theoretical models are often employed that preserve the essential features of QCD while allowing for more tractable analytical treatment. Among these, the Abelian Higgs model plays a prominent role, as it incorporates scalar and gauge fields in a manner analogous to the Higgs mechanism of the Standard

Model and serves as a powerful framework for exploring non-perturbative phenomena such as confinement, magnetic monopole formation, dyon dynamics and dual superconductivity.

Within this framework, dyon condensation refers to the collective emergence of dyons hypothetical excitations carrying both electric and magnetic charges which are central to understanding the non-perturbative structure of the QCD vacuum. Dyons provide a natural mechanism for addressing the confinement problem, wherein quarks and gluons are permanently bound into color-neutral hadronic states. The study of dyon condensation sheds light on the formation of topological defects, including magnetic monopoles and reveals deeper aspects of the strong interaction at low energies. Closely related to this phenomenon is the concept of dual superconductivity, which offers a compelling picture of confinement in QCD. In this scenario, the QCD vacuum behaves as a dual superconductor, expelling color-electric flux in a manner analogous to the Meissner effect in conventional superconductors, but operating in a dual (magnetic) formulation. This dual behavior naturally leads to the formation of flux tubes that confine quarks and gluons. A detailed understanding of dual superconductivity not only strengthens our insight into the structure of the QCD vacuum but also provides a valuable framework for exploring the phase structure of strongly interacting matter, with significant implications for high-energy physics, early-universe cosmology and the physics of dense astrophysical systems.

**1.1 Background on Quantum Chromodynamics (QCD)**

Quantum Chromodynamics (QCD) is the fundamental theory governing the strong nuclear interaction, which binds quarks into hadrons such as protons and neutrons. Although QCD has been highly successful in describing particle interactions at high energies, a comprehensive understanding of its behavior under extreme conditions such as those present in the early universe or within dense astrophysical environments remains a significant theoretical challenge. Investigating QCD in these regimes is crucial for deepening our knowledge of fundamental particle physics and cosmology. Progress in this direction promises valuable insights into the nature of strongly interacting matter, the evolution of the universe and the fundamental forces that shape it.

**1.2 Dyon Condensation**

Within the Abelian Higgs framework, dyon condensation arises from the spontaneous formation of dyons hypothetical excitations that carry both electric and magnetic charges. This mechanism offers important insight into the non-perturbative dynamics of the model and provides a coherent explanation for phenomena such as quark confinement and the emergence of magnetic monopoles. By examining dyon condensation in detail, researchers seek to uncover the underlying mechanisms that govern strongly interacting systems, thereby advancing our understanding of fundamental particle physics and the deeper structure of the universe.

## 2.REVIEW OF LITERATURE

**Cardinali, M., D'Elia, M., & Pasqui, A. (2021)** This study investigates thermal monopole condensation within the framework of Quantum Chromodynamics (QCD), with particular emphasis on the influence of physical quark masses. Using lattice QCD simulations, the research analyzes the behavior of monopole currents at finite temperature and density. The results provide important insight into the role of monopoles in the non-perturbative dynamics of QCD, especially in connection with confinement mechanisms and phase transitions. By examining QCD under extreme conditions, this work contributes significantly to a deeper understanding of the strong interaction and the fundamental behavior of matter in highly energetic environments.

**Gunkel, P. J. (2021).** This study examines Quantum Chromodynamics (QCD) in the presence of Gribov copies, with particular focus on the structure of the BRST (Becchi–Rouet–Stora–Tyutin) invariant vacuum. It analyzes how Gribov ambiguities influence BRST symmetry and modify the vacuum properties of QCD. Employing a combination of analytical and numerical techniques, the work elucidates the consequences of Gribov copies for non-perturbative phenomena such as color confinement and chiral symmetry breaking. These findings offer valuable insight into the complex nature of the QCD vacuum and underscore its significance for theoretical developments in high-energy physics.

**Gunkel, P. J. (2021)** This study investigates the influence of hadronic degrees of freedom on the Quantum Chromodynamics (QCD) phase diagram, with particular emphasis on their interaction with the quark–gluon plasma phase. By employing a combination of lattice QCD simulations and effective theoretical models, the research examines the thermodynamic properties of QCD matter over a wide range of temperatures and densities. The results highlight the essential role of hadronic contributions in shaping the phase structure of QCD, especially in relation to phase transitions. Moreover, the study provides important insights into the behavior of strongly interacting matter under extreme conditions, offering significant implications for both high-energy physics and astrophysical applications.

**Ihssen, F. J. (2023)** Quantum Chromodynamics (QCD) is the fundamental theory governing the strong nuclear interaction. This doctoral research investigates the phase structure of QCD with the objective of elucidating the complex nature of phase transitions and critical phenomena in strongly interacting matter. By integrating theoretical frameworks with numerical simulations and analytical methods, the study provides an in-depth examination of QCD behavior across varying physical conditions. The analysis focuses in particular on transitions between distinct phases of QCD matter, especially the transformation from hadronic matter to the quark–gluon plasma. Through a detailed exploration of the QCD phase diagram, this work advances our understanding of the fundamental properties of QCD and its dynamics under extreme conditions, with significant

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implications for high-energy physics, early-universe cosmology and the physics of dense astrophysical systems.

**Issifu, A., & Brito, F. A. (2021).** Within the framework of Quantum Chromodynamics (QCD), this study develops an effective theoretical model to describe glueballs bound states composed purely of gluons and the emergence of dual superconductivity at finite temperature. Employing an effective field theory approach, the authors construct a model that captures the essential dynamics governing glueball formation and dual superconducting behavior in QCD. Through a combination of analytical methods and numerical simulations, the research examines the thermodynamic properties and phase transitions associated with these phenomena. The findings provide valuable insight into the non-perturbative regime of QCD, improving our understanding of strongly interacting matter under thermal constraints. Moreover, the proposed model offers a robust framework for future theoretical investigations and experimental studies of QCD matter at finite temperatures, with wide-ranging implications for high-energy physics and strongly coupled systems.

## 3. DYNAMIC INTERACTION AND ELECTROMAGNETIC DUALITIES

We construct a measure-invariant and Lorentz-covariant quantum field–theoretic formulation for dyonic fields within a simplified theoretical framework. The model introduces two four-potentials and explicitly incorporates the complex nature of physical quantities such as total charge, total current and the generalized four-potential, each comprising real (electric) and imaginary (magnetic) components. This unified formulation provides a consistent relativistic and quantum description of electric and magnetic interactions, allowing both sectors to be treated within a single coherent theoretical framework.

generalized charge

$$q = e - ig,$$

generalized four-current

$$J_\mu = j_\mu - ik_\mu,$$

Further, the generalized four-potential

$$V_\mu = A_\mu - iB_\mu,$$

In this formulation, jμj^\mujμ and kμk^\mukμ represent the electric and magnetic four-current densities, respectively, while AμA^\muAμ and BμB^\muBμ denote the corresponding electric and magnetic four-potentials associated with dyonic fields. The parameters eee and ggg correspond to the electric and magnetic charges of the dyons. Using the wave function of the combined (generalized) field, we proceed to describe the dynamics of the system as follows:

$$\vec{\varphi} = \vec{E} - i\vec{H},$$

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$$\vec{\nabla} \cdot \vec{\Psi} = J_0,$$

$$\vec{\nabla} \times \vec{\Psi} = -i\vec{J} - i\frac{\partial \vec{\Psi}}{\partial t},$$

These fields' generalized field equations can be expressed as
where 2. According to 1b, the spatial and temporal components of Jμ.
For these equations, the compact form can be expressed as

$$G_{\mu\nu,\nu} = J_\mu,$$

$$G^d_{\mu\nu,\nu} = 0,$$

where Gμν the generalized field tensor, is given as

$$G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

and Gd μν is its dual given as

$$G^d_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}.$$

Equation 2.4 may also be written as

$$G_{\mu\nu} = F_{\mu\nu} - iH_{\mu\nu},$$

Where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

Then, at that point, 2.3reduces to the accompanying structure:

$$F_{\mu\nu,\nu} = j_\mu,$$

$$H_{\mu\nu,\nu} = k_\mu.$$

Under the duality transformations, these equations are symmetrical.

$$F_{\mu\nu} \longrightarrow H_{\mu\nu}, \qquad H_{\mu\nu} \longrightarrow -F_{\mu\nu}, \qquad j_\mu \longrightarrow k_\mu, \qquad k_\mu \longrightarrow -j_\mu.$$

The all-encompassing fee The density of the Lagrangian for spin-1 This is how the bosonic dyon} with rest mass m0 is described in the Abelian theory:

$$\begin{aligned} L &= m_0 - \frac{1}{4}\Big[\alpha\Big\{(A_{\nu,\mu} - A_{\mu,\nu})^2 - (B_{\nu,\mu} - B_{\mu,\nu})^2\Big\} - 2\beta\{(A_{\nu,\mu} - A_{\mu,\nu})(B_{\nu,\mu} - B_{\mu,\nu})\}\Big] \\ &\quad + \{(\alpha A_\mu - \beta B_\mu)j_\mu - (\alpha B_\mu + \beta A_\mu)k_\mu\} \\ &= L_P + L_F + L_I, \end{aligned}$$

**4. TWOFOLD SUPERCONDUCTIVITY VIA THE ENHANCED MEISSNER EFFECT**

Instead of considering dyonic particles, we focus on distinct species that carry either electric or magnetic charges exclusively. Under this assumption, the field equations given in Eq. (2.3) simplify to the following form:

$$F_{\mu\nu,\nu} = j_\mu,$$

$$F^d_{\mu\nu,\nu} = 0,$$

$$H_{\mu\nu,\nu} = k_\mu,$$

$$H^d_{\mu\nu,\nu} = 0$$

or then again equally

$$A_\mu = j_\mu,$$

$$B_\mu = k_\mu,$$

and equation of motion 2.12 becomes

$$m\ddot{x}_\mu = (eF_{\mu\nu} + gH_{\mu\nu})u^\nu.$$

Upon modifying these equations, dual invariance naturally emerges. Consequently, the effective action within the Abelian-projected formulation of QCD can be written as follows:

$$S = -\frac{1}{4}\int F_{\mu\nu}(x) \in (x-y) F^{\mu\nu}(y) d^4x d^4y - \frac{1}{4}\int H_{\mu\nu}(x)\mu(x-y)H^{\mu\nu}(y)d^4xd^4y + j_\mu A^\mu + k_\mu B^\mu.$$

Then, the current-correlations might be expressed like this:

$$\langle j_\mu \rangle = \frac{\delta S}{\delta A_\mu}, \qquad \langle k_\mu \rangle = \frac{\delta S}{\delta B_\mu},$$

$$\langle j_\mu(x) j_\nu(y) \rangle = \frac{\delta^2 S}{\delta A_\nu(y)\delta A_\mu(x)},$$

$$\langle k_\mu(x) k_\nu(y) \rangle = \frac{\delta^2 S}{\delta B_\nu(y)\delta B_\mu(x)}.$$

Regarding the activity in question, these relationships result in

$$\langle j_\mu(x) j_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4}\left[k^2\delta_{\mu\nu} - k_\mu k_\nu\right] \in \left(k^2\right),$$

$$\langle k_\mu(x) k_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4}\left[k^2\delta_{\mu\nu} - k_\mu k_\nu\right]\mu\left(k^2\right).$$

For minor alterations, we possess

$$\in\left(k^2\right)=1\pm\chi_e\left(k^2\right),$$
$$\mu\left(k^2\right)=1\mp\chi_g\left(k^2\right),$$

Here, the upper signs on the right-hand side correspond to vacuum polarization effects arising from charged particle loops, whereas the lower signs represent contributions from monopole loops. Relation 3.5~ is another way to write it.

$$\langle j_\mu(x)j_\nu(y)\rangle=-\int\frac{d^4k}{(2\pi)^4}\left[\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right]m_{L_e}^2,$$

$$\langle k_\mu(x)k_\nu(y)\rangle=-\int\frac{d^4k}{(2\pi)^4}\left[\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right]m_{L_g}^2.$$

According to these relations, the AμA_\muAμ-propagator experiences a screening effect due to charged particle loops (εk2≥1)(\varepsilon k^2 \geq 1)(εk2≥1), whereas the BμB_\muBμ-propagator undergoes an antiscreening effect associated with the corresponding photon acquiring a mass mLem_{Le}mLe. In contrast, monopole loops induce an antiscreening effect in the AμA_\muAμ-propagator and a screening effect in the BμB_\muBμ-propagator, accompanied by the generation of a photon mass mLgm_{Lg}mLg. As a result, the dual fields BμB_\muBμ coupled to electric charges and AμA_\muAμ coupled to monopoles are weakly interacting and become antiscreened by charged particles and monopoles, respectively, while each species screens its own direct potential. The combined Meissner effect arising from this dual antiscreening mechanism leads naturally to the emergence of dual superconductivity.

**5. DYON CONFINEMENT AND CONDENSATION**

The non-Abelian nature of the gauge groups SU(3)SU(3)SU(3) or SU(2)SU(2)SU(2) plays a central role in the confinement mechanism associated with dyon condensation. One of the most effective theoretical approaches for addressing confinement in non-Abelian gauge theories is the Abelian projection method. In general, dyons are intrinsically non-Abelian objects, possessing both a conventional four-dimensional spacetime structure and an internal gauge space of dimension nnn. The corresponding dyonic fields therefore exhibit internal degrees of freedom, transforming according to the adjoint representation of the underlying non-Abelian gauge symmetry group. By selecting SU(2)SU(2)SU(2) as the internal gauge group, the total dyonic field tensor can be explicitly constructed, providing a tractable framework for analyzing dyon dynamics and confinement within the Abelian-projected theory.

$$\vec{G}_{\mu\nu}=G^a_{\mu\nu}T_a$$

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using the definition of the generalized four-potential as

$$\vec{V}_\mu = V^a_{\mu\nu} T_a,$$

Here, the vector notation denotes components in the internal gauge space, repeated indices run over the internal degrees of freedom a=1,2,3a = 1, 2, 3a=1,2,3 and the matrices TaT^aTa represent the generators of the SU(2)SU(2)SU(2) gauge group, satisfying the corresponding commutation relations.

$$[T_a, T_b] = i\varepsilon_{abc} T_c$$

where εabc is the structure constant of the internal group. The non-Abelian version of 2.4\ that follows can be used to connect G μv and V\.:

$$G^a_{\mu\nu} = \partial_\nu V^a_\mu - \partial_\mu V^a_\nu + |q| \varepsilon^{abc} V_{\mu b} V_{\nu c},$$

Here, the generalized dyonic charge is defined as in Eq. (1a). With an appropriate choice of the Lagrangian density, one can obtain classical dyonic solutions of a spontaneously broken non-Abelian SU(2)SU(2)SU(2) gauge theory.

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2} (D_\mu \phi)^a (D^\mu \phi)_a - V(\phi)$$
$$= L_{\text{dyon}}(A_\mu, B_\mu, \phi), \quad \text{where } D_\mu \phi = \partial_\mu \phi - i \operatorname{Re}(q * V_\mu) \phi = (\partial_\mu - ieA_\mu - igB_\mu) \phi,$$

Re stands for the actual portion and

$$V(\phi) = \frac{1}{4} (\phi^a \phi_a)^2 - \frac{1}{2} v^2 (\phi^a \phi_a) \quad \text{with } v = \langle \phi \rangle = \langle 0 | \phi | 0 \rangle$$

Finding the Higgs field's vacuum expectation value. Putting this equation simply, it can be expressed as

$$V(\phi) = -\eta \left( |\phi|^2 - v^2 \right)^2$$

using the constant η.

The gauge-dependent component of the Lagrangian, the first term of rhs in 4.5}, remains unchanged when the fields Aμ and Bμ undergo the following transformations:

$$V_\mu = \begin{bmatrix} A_\mu \\ B_\mu \end{bmatrix} \longrightarrow \begin{bmatrix} A'_\mu \\ B'_\mu \end{bmatrix} = V_\mu' = R(\delta) \begin{bmatrix} A_\mu \\ B_\mu \end{bmatrix} = R(\delta) V_\mu, \quad \text{where } R(\delta) = \begin{bmatrix} \cos\delta \sin\delta \\ -\sin\delta \cos\delta \end{bmatrix}$$

With

$$\delta = \tan^{-1}\left(\frac{g}{e}\right).$$

The electric and magnetic fields of dyons can be calculated using the following approach, given the Lagrangian density.

$$V_{ia} = \varepsilon_{aij}(\vec{r})^{j}\frac{K(r)-1}{|q|r^{2}},$$

$$V_{0a} = (\vec{r})_{a}\frac{J(r)}{|q|r^{2}},$$

$$\phi_{a} = (\vec{r})_{a}\frac{H(r)}{|q|r^{2}},$$

where the following equations are satisfied by the functions Kr~, Jr~ and Hr}:

$$r^{2}H''(r) = 2HK^{2},$$

$$r^{2}J''(r) = 2JK^{2},$$

$$r^{2}K''(r) = K\left(K^{2}-1\right) + K\left(H^{2}-J^{2}\right).$$

The following could be used to express a solution to these equations:

$$J(r) = \tilde{\alpha}\phi(r), \quad H(r) = \tilde{\beta}\phi(r), \quad K(r) = \frac{Cr}{\sinh Cr},$$

$$\text{where } \tilde{\beta}^{2} - \tilde{\alpha}^{2} = 1, \; \phi(r) = C(r)\coth Cr - 1.$$

In the limit of Prasad-Sommerfield

$$V(\phi) = 0; \quad \text{but } v = \langle\phi\rangle \neq 0.$$

The dyons have the lowest energy in this limit for given magnetic and electric charges, g and e, respectively. As a result, we obtain the following dyonic mass expression:

$$M = v\left(e^{2}+g^{2}\right)^{1/2} = v|q|,$$

where the first-order equations apply to the electric and magnetic fields connected to dyons.

$$E_{i}^{a} = G_{0i}^{a} = \partial^{i}V_{0}^{a} + |q|\varepsilon^{abc}V_{ib}V_{0c} = \left(D_{i}\phi\right)^{a}\sin\alpha,$$

$$B_{i}^{a} = \varepsilon_{ijk}G^{jka} = \left(D_{i}\phi\right)^{a}\cos\alpha, \qquad \text{where } \alpha = \tan^{-1}\frac{e}{g}.$$

$$D_{0}(\phi)^{a} = 0,$$

Here, AAA denotes an SU(2)SU(2)SU(2) vector index, while iii and 000 represent the spatial and temporal components, respectively. The electric and magnetic fields associated with dyons possess both internal (gauge) and external (spacetime) components, reflecting their inherently non-Abelian character. To obtain the Abelian projection, we impose the following condition:

$$K(r) \rightarrow 0, \qquad J(r) \rightarrow b + cr,$$

where b and c are positive constants representing the mass and charge elements, respectively. These fields dampen the corresponding structure to the best of their abilities:

$$E_j^a = -\frac{3b}{|q|r^4}(\vec{r})^a(\vec{r})_j - \frac{2c}{|q|r^3}(\vec{r})^a(\vec{r})_j,$$

$$B_j^a = -\frac{(\vec{r})_j(\vec{r})^a}{|q|r^4}.$$

The electric and magnetic charges of point-like, massless dyons are determined by the corresponding field configurations in the theory. These electric fields exhibit the characteristic behavior of positively charged dyons, with the electric charge proportional to 3b|q|3b|q|3b|q| and the magnetic charge proportional to |q||q||q|, as the parameter ccc decreases and ultimately vanishes. In this limit, non-Abelian dyons effectively reduce to Abelian dyons through the Abelian projection procedure. The Abelian Higgs Model (AHM), which incorporates dyon condensation, provides a widely used theoretical framework for probing the infrared properties of Quantum Chromodynamics within the Abelian projection. In this approach, a complex scalar field ϕχ\phi_\chiϕχ represents the dyon field, while two massive gluons are denoted by Wμ±W_\mu^{\pm}Wμ±, along with a U(1)U(1)U(1) gauge field associated with the combined Abelian sector. Consequently, the Lagrangian density given in Eq. (4.5) simplifies significantly in this framework, enabling a clearer understanding of dyonic interactions and their implications for fundamental physical phenomena such as confinement and infrared dynamics in QCD.

$$L_{\text{dyon}}(A_\mu, B_\mu, \phi) = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\left|(\partial_\mu - ieA_\mu - igB_\mu)\phi\right|^2 + \eta\left(|\phi|^2 - v^2\right)^2.$$

The parcel capability in the Euclidean space-time can be communicated as far as this Lagrangian as

$$Z_{\text{dyon}} = \int DA_\mu DB_\mu D\phi \exp\left\{-\int d^4x L_{\text{dyon}}(A_\mu, B_\mu, \phi)\right\}.$$

Integrating over the field A and applying the transformation 4.8}
However, this partitioning algorithm
Reductions in AHM to this kind:

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$$Z_{\text{dyon}} = \int DB'_\mu D\phi \exp\left\{-\int d^4x L_{\text{AHM}}\left(B'_\mu, \phi\right)\right\},$$

$$\text{with } L_{\text{AHM}}\left(B'_\mu, \phi\right) = -\frac{1}{4}H'_{\mu\nu}H'^{\mu\nu} + \frac{1}{2}\left|\left(\partial\mu - i\overline{g}B'_\mu\right)\phi\right|^2 + \eta\left(|\phi|^2 - v^2\right)^2,$$

where the Higgs field's magnetic charge resides φ

$$\overline{g} = |q|, \qquad H'_{\mu v} = \partial_\mu B'_v - \partial_v B'_\mu.$$

This model AHM combines dual superconductivity and control due to dyonic condensation since the Higgs-type mechanism arises here.

**6. DYONIC LOOP IN ABELIAN HIGGS MODEL**

Notably, the partition function defined in Eq. (4.19) of the dyonic theory can be identified with the quantum expectation value of the Wilson loop, a central object in gauge field theories. The Wilson loop encodes the phase acquired by an electrically charged particle traversing a closed contour in spacetime and serves as a fundamental probe of the gauge structure of the theory. Within the context of dyonic interactions, this expectation value offers critical insight into the dynamics of both electric and magnetic charges. An examination of the Wilson loop in this framework elucidates key non-perturbative phenomena, including confinement, dual superconductivity and the behavior of dyonic fields. Consequently, the Wilson loop expectation value emerges as a powerful diagnostic tool for investigating the underlying electromagnetic and topological structure of dyonic gauge theories.

$$\langle W_l^c \rangle_{\text{dyon}} = \frac{1}{Z_{\text{dyon}}} \int DA_\mu DB_\mu D\phi \exp\left\{-\int d^4x L_{\text{dyon}}(A_\mu, B_\mu, \phi)\right\} W_l^c(A_\mu),$$

Were

$$W_l^c(A_\mu) = \exp\left\{ie_0 \int d^4x \eta_\mu A^\mu\right\}$$

With

$$\eta_\mu(x) = \oint_C d\tilde{x}_\mu \, \delta^{(4)}(x - \tilde{x}(\tau)),$$

This construction generates a particle carrying electric charge e0e_0e0 along the world trajectory CCC. Applying the transformation given in Eq. (4.8) to the quantum expectation value in Eq. (5.1a) and subsequently integrating over the gauge field A~μ\tilde{A}_\muA~μ, we obtain an expression involving the Wilson loop WCW_CWC and the ’t Hooft loop TCT_CTC, which appear as components of the composite operator KCqe,qmK_C^{q_e,q_m}KCqe,qm.

$$\langle W_l^c \rangle_{\text{dyon}} = \left\langle K^c_{(q_l, q_m)}\left(B'_\mu\right)\right\rangle_{\text{AHM}}$$

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$$K^{c}_{(q_e,q_m)}\left(B'_{\mu}\right) = H^{c}_{q_e}\left(B'_{\mu}\right) \cdot W^{c}_{q_m}\left(B'_{\mu}\right), \quad \text{where } q_e = \frac{e_0 g}{|q|},\ q_m = \frac{e_0 e}{|q|}.$$

The effective four-current density of electric and magnetic energy can be expressed as follows:

$$j_{\mu} = q_e \eta_{\mu}, \qquad k_{\mu} = q_m \eta_{\mu}.$$

The operator Hc qe B μ is

$$H^{c}_{q_e}\left(B'_{\mu}\right) = \exp\left\{-\frac{1}{4}\int d^4x\left[\left(H'_{\mu v} - \frac{1}{2}\varepsilon_{\mu v\alpha\beta}F_{\alpha\beta}\right)^2 - H'_{\mu v}H'^{\mu v}\right]\right\}, \quad \text{where } H'_{\mu v} = \partial_{\mu}B'_{v} - \partial_{v}B'_{\mu}$$

The dual field tensor, denoted by F~αβ\tilde{F}_{\alpha\beta}F~αβ, exhibits important properties within the theoretical framework under consideration. In classical electromagnetism, the field tensor FαβF_{\alpha\beta}Fαβ provides a compact representation of both electric and magnetic fields. In more advanced theoretical settings particularly those involving dyons or non-Abelian gauge structures it becomes necessary to extend this formulation. In such cases, the dual field tensor plays a central role in describing the coupling and interplay between electric and magnetic charges, thereby offering a more complete characterization of the underlying field dynamics. Constructed directly from the original field tensor, F~αβ\tilde{F}_{\alpha\beta}F~αβ serves as a fundamental mathematical object that facilitates an alternative and more comprehensive interpretation of electromagnetic interactions, especially in regimes where conventional formulations are insufficient.

$$F_{\mu v,v} = j_{\mu}$$

In this context, Eq. (2.7a) closely parallels the standard electrodynamic field tensor associated with Abelian dyons. The equation effectively expresses the field tensor in terms of its dual counterpart and the operator HcH_cHc, which are linked to the Wilson loop and the 't Hooft loop, respectively. This formulation indicates that, within the Abelian dyon framework, the dual field tensor and the associated loop operators provide an equivalent and comprehensive description of the electromagnetic field tensor, capturing the dynamics of electromagnetic interactions mediated by dyons. The correspondence between the field expressions derived from the Abelian dyon theory and those of conventional electrodynamics highlights the internal consistency of the model and reinforces its effectiveness in describing electromagnetic phenomena from a dual and extended theoretical perspective.

**7.CONCLUSION**

Within the framework of quantum field theory, this work presents a comprehensive theoretical investigation of dyonic interactions and electromagnetic duality. The analysis begins with the formulation of a Lorentz-covariant quantum field theory for dyons based on a group-theoretical approach, incorporating generalized charges, currents and gauge potentials that account for both electric and magnetic components. The construction of the corresponding Lagrangian densities

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and symmetric field equations demonstrates the invariance of the theory under duality transformations, thereby establishing its internal consistency. The discussion subsequently addresses the emergence of dual superconductivity through a generalized Meissner effect, illustrating how charged particles and magnetic monopoles respectively induce screening and antiscreening of dual gauge potentials, leading to a dual superconducting phase. Extending the framework to non-Abelian gauge theories, particular emphasis is placed on SU(2)SU(2)SU(2) as the internal gauge symmetry. Employing the Abelian projection technique, the analysis provides insight into confinement mechanisms arising from dyon condensation within the Abelian Higgs model. Furthermore, the study of dyonic loop configurations offers additional understanding of confinement dynamics and the role of dyonic interactions. Overall, this work delivers a detailed and coherent exposition of the theoretical foundations of dyonic fields and electromagnetic duality, highlighting their significant implications for dual superconductivity and confinement phenomena in contemporary quantum field theory.

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