# LINEAR ALGEBRA AS THE MATHEMATICAL FOUNDATION OF ARTIFICIAL INTELLIGENCE: CONCEPTS, APPLICATIONS, AND FUTURE PROSPECTS

**Mrs. Kavita Rani**
**Assistant Professor, Mathematics**
**Govt. College, Raipur Rani, Panchkula**

## ABSTRACT

Linear algebra, a fundamental tool in the field of mathematics, plays an essential part in the mathematical foundation of the groundbreaking arena of Artificial Intelligence (AI). This mathematics branch offers immense possibilities for capturing complex relationships within datasets through matrices and vectors. These mathematical tools facilitate the concise representation of large volumes of data, contributing to efficient data analysis between them, bringing immense value to AI algorithms and models. This research paper investigates the pivotal role of linear algebra as the fundamental mathematical framework underpinning artificial intelligence. This paper also explores the foundational concepts of linear algebra and its direct applications in various AI algorithms, demonstrating how matrices, vectors, and transformations provide the essential mathematical scaffolding for the development and optimization of intelligent systems.

**Keywords**: Linear Algebra, Matrices, Vectors, Artificial Intelligence, Foundation

## 1. INTRODUCTION

Artificial Intelligence (AI) is a branch of computer science that creates systems to perform tasks that typically require human brain functioning or intelligence (Pannu, 2015). This includes problem-solving, learning, understanding natural language, and adapting to new situations. AI encompasses a range of techniques for example machine learning, neural networks, and natural language processing, aims to develop artificial system that can mimic cognitive functions. Applications of AI are diverse, spanning from virtual assistants and image recognition to autonomous vehicles and advanced decision-making systems. (Nascimento, et al. 2019).

In artificial intelligence (AI), linear algebra plays a pivotal role as a mathematical framework for representing and manipulating data. Vectors and matrices facilitate the description of features, transformations, and relationships within datasets. Linear algebra operations, such as matrix multiplication, eigenvalue decomposition, and vector calculus, are fundamental in various machine learning algorithms (Dhanalakshmi, P. 2021). Techniques like Principal Component Analysis (PCA) and Singular Value Decomposition (SVD) leverage linear algebra for dimensionality reduction and feature extraction. Overall, linear algebra provides the mathematical foundation essential for modeling complex AI problems and enhancing computational efficiency in data processing and analysis (Olver, P. J. ,et al 2006).

## Basic concepts of Linear Algebra in AI

### 1. (i) Vector Spaces and Data Representation

A vector space is a fundamental concept in linear algebra that provides a framework for studying and understanding linear relationships and structures (LAX, P.D. 2007) and mathematical structure consisting of a set of elements called vectors, along with operations of addition and scalar multiplication.A vector space V over a field F (usually the real numbers or complex numbers) must satisfy the following properties:

a. Closure under Addition: The sum of any two vectors in the space is also in the space. For a,b Ɛ V, a + b ƐV
b. Closure under Scalar Multiplication: When a vector is multiplied by a scalar (a real or complex number), the result is still in the vector space. For u Ɛ F, a Ɛ V, ua Ɛ V
c. Associativity of Addition: The order in which vector additions are performed does not affect the resultfor all a,b,c Ɛ V, a + (b + c) =(a + b) + c
d. Commutativity of Addition:The addition of vectors is commutative.For all a, b Ɛ V, a + b Ɛ V
e. Identity Element of Addition: There exists a zero vector such that adding it to any vector leaves the vector unchanged. For all a Ɛ V there 0 Ɛ V such that
   a + 0 = a = 0 + a
f. Inverse Elements of Addition: For every vector, there exists a unique additive inverse (negation) such that adding it to the original vector yields the zero vector. For all a Ɛ V there exist –a Ɛ V such that a + (-a) = 0 = (-a) + a
g. Compatibility of Scalar Multiplication with Field Multiplication: Scalar

multiplication distributes over field multiplication. For u,v Ɛ V, a, b Ɛ v u(a + b) = ua + ub and (u+v)a = ua + va

h. Identity Element of Scalar Multiplication: Multiplying a vector by the field's multiplicative identity leaves the vector unchanged. for 1 Ɛ F, a Ɛ V
1 a =a = a 1

### (ii) Subspaces:

A subspace W of a vector space V(F) is a subset that is itself a vector space with the inherited vector space operations. Subspaces must satisfy the properties of closure under vector addition and scalar multiplication.

### (iii)Basis and Dimension:

A basis B for a vector space is a set of vectors that spans and is linearly independent.

The dimension of a vector space is the number of vectors in any basis.

### (iv) Data Representation

Vector spaces play a crucial role in representing features, data points, and model parameters in artificial intelligence. Here's how:

a. Vectorization of Features: Features of data points are often represented as vectors. Each feature corresponds to a component in the vector, allowing the entire set of features to be represented as a single vector. This simplifies the mathematical representation and manipulation of features (Bellet, et al., 2013).
b. Data Vectorization: Individual data points in a dataset are commonly represented as vectors in a vector space. Each element of the vector corresponds to a specific attribute or feature of the data point. Vectorizing data enables efficient storage, manipulation, and analysis.
c. Model Parameters: In machine learning models, especially linear models, the parameters (weights) are often represented as vectors. The dot product of the feature vector and the weight vector forms the basis of many algorithms, representing the strength and influence of each feature (Suthaharan, 2016).

These properties make vector spaces essential in AI, particularly in tasks involving linear transformations, linear equations, and data representation (Najafabadiet al., 2015). Linear algebra, rooted in vector space theory, underlies many AI algorithms, including those used in machine learning and deep learning. Vectors and matrices provide a concise and efficient way to represent and process data in various AI applications.

Thus, vector spaces provide a versatile and efficient framework for representing features, data points, and model parameters in artificial intelligence. This representation facilitates mathematical operations, transformations, and computations fundamental to various AI algorithms.

## 2. (i) MATRICES AND ITS OPERATIONS:

Matrices are rectangular arrays of numbers, symbols, or expressions arranged in rows and columns. The size is specified by the number of rows and columns, like "m x n" where m is the number of rows and n is the number of columns. Elements in a matrix are often denoted by letters with subscripts, such as Aij, where i represents the row and j represents the column (Eves, 1980).

$A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ represent matrix of order 2 x 2 having 2 rows and 2 columns.

(ii) Matrix Operations:

Addition and Subtraction: Matrices of the same size can be added or subtracted by performing the operation elementwise.

Scalar Multiplication: A matrix can be multiplied by a scalar by multiplying each element by that scalar.

Matrix Multiplication: is a crucial operation in linear algebra. If A is an m x n matrix and B is an n x p matrix, their product AB is an m x p matrix.The product's entry in the i-th row and j-th column is the sum of the products of corresponding elements from the i-th row of A and the j-th column of B. Matrix multiplication is not commutative, meaning AB is not necessarily equal to BA.

(iii)Identity and Inverse Matrices:

The identity matrix, denoted as I, is a square matrix with ones on its main diagonal and zeros elsewhere. For any matrix A of appropriate size, AI = A = IA.If a square matrix A has an inverse (denoted as $A^{-1}$), the product of A and its inverse is the identity matrix: $A^{-1}A = AA^{-1} = I$.

(iv)Determinant:

The determinant of a square matrix is a scalar value that can provide information about the matrix's properties.

For a 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant is ad - bc.

(v)Transpose:The transpose of a matrix is obtained by swapping its rows with columns. If A is an m x n matrix, its transpose, denoted as A^t, is an n x m matrix.

(vi)Matrix Representation of Linear Systems:

Matrices are often employed to represent systems of linear equations. The coefficients of the variables form a matrix, and the constants on the right side of the equations constitute a column vector. The system can then be represented as the matrix equation Ax = b, where A is the coefficient matrix, x is the column vector of variables, and b is the constant vector. As

$2x + 3y = 5$

$7x + 8y = 9$ this system of equation can be written as AX=B

$$\begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

These matrix concepts are foundational for solving linear systems, analyzing transformations, and understanding the structure of vector spaces in linear algebra. They have wide applications in various fields, including physics, computer science, and engineering (Meyer, C. D. 2023).

## 3. EIGEN VALUES AND EIGEN VECTORS

Eigenvalues and eigenvectors are important concepts in linear algebra, often used to analyze and understand linear transformations represented by matrices. Let's delve into these concepts:

(i) Eigenvalues (λ): Eigenvalues are scalar values that characterize how a linear transformation (represented by a matrix) stretches or compresses space along its principal axes.For a square matrix A, an eigenvalue λ satisfies the equation det (A - λI) = 0, where I is the identity matrix. This equation is called the characteristic equation.The solutions to the characteristic equation are the eigenvalues of the matrix.

(ii) Eigenvectors (v): Eigenvectors are non-zero vectors that remain in the same direction after a linear transformation represented by a matrix, only scaled by their corresponding eigenvalue.For an eigenvalue λ, the eigenvectors v satisfy the equation (A - λI)v = 0. These vectors are in the null space of the matrix (A - λI). Eigenvectors are not unique; any scalar multiple of an eigenvector is also an eigenvector.

(iii) Geometric Interpretation: Eigenvalues determine the scale factor by which the eigenvectors are stretched or compressed during a linear transformation. If a matrix A has distinct eigenvalues, its eigenvectors form a set of linearly independent vectors, providing a basis for the vector space.

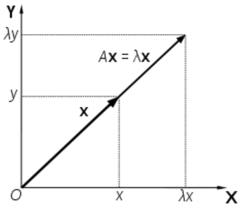


Fig 1. Geometric representation of Eigen value and eigen vector

*Source*: https://www.google.com/url?sa=i&url=https%3A%2F%2Fen.wikipedia.org%2Fwiki%2FEigenvalues_and_eigenvectors

(iv) Diagonalization: A square matrix A can be diagonalized if it has n linearly independent eigenvectors, forming a matrix P. The diagonal matrix D is composed of the corresponding eigenvalues: A = PDP', where D is a diagonal matrix with eigenvalues on the main diagonal.Understanding eigenvalues and eigenvectors provides valuable insights into the behavior of linear transformations represented by matrices and plays a crucial role in many mathematical and computational applications (Golubet al., 2000).

## 4. LINEAR TRANSFORMATIONS

A linear transformation is a mathematical function between two vector spaces that preserves the structure of vector addition and scalar multiplication. In other words, it maps vectors to vectors in a way that reflects the linear properties of the underlying vector spaces.

(i) Definition: LetV(F) and W(F) be vector spaces over the same field F. A function T: V→ W is a linear transformation if, for all vectors u, v in Vand all scalars c in F, the following two conditions hold:

T (u + v) = T(u) + T(v)) (Preservation of vector addition)

T(cu) = cT(u) (Preservation of scalar multiplication)

(ii) Matrix Representation: Every linear transformation can be represented by a matrix. The columns of the matrix are the images of the standard basis vectors of T.

(iii) Kernel and Image (or Range): The kernel (null space) of a linear transformation is the set of all vectors in the domain that map to the zero vector in the codomain ker(T) = {v in V: T(v) = 0}. The image (or range) of T is the set of all vectors in the codomain that have at least one preimage in the domain: img(T) = {w in W: there exists v in V, T(v) = w}. Linear transformations preserve linear combinations: The composition of linear transformations is also a linear transformation. The identity transformation is a linear transformation. The inverse of a bijective linear transformation is also a linear transformation.

(iv) Matrix Transformations: Multiplying a vector by a matrix represents a linear transformation.

- Rotation and Scaling: Certain geometric transformations like rotations and scaling are linear transformations.
- Projection: The projection of a vector onto a subspace is a linear transformation.

Linear transformations are widely used in various fields, including computer graphics, image processing, quantum mechanics, and data analysis. In computer graphics, for instance, linear transformations are used to rotate, scale, and translate objects in a three-dimensional space (Velhoet al., 2009). Understanding linear transformations is crucial in linear algebra, providing a way to analyze and manipulate vector spaces and their structures. Matrices, which are higher-dimensional analogs of vectors, are used to represent linear transformations. In machine learning, transformations of data or features are often achieved through matrix multiplication, providing an efficient way to represent complex relationships (Srebro, 2004)

## 5. APPLICATIONS OF LINEAR ALGEBRA IN AI

5.1 **Neural Network:** Neural networks rely heavily on linear algebra concepts. At their core, they consist of layers of interconnected nodes, or neurons, where each connection is associated with a weight (Mehrotra et al., 1997). Linear algebra operations, such as matrix multiplication and activation functions, play crucial roles in the functioning of neural networks. The input, weights, and biases can be represented as matrices and vectors (Zhangetal., 2021). The output of a neuron is often computed through the dot product of input values and corresponding weights, followed by the addition of bias. Mathematically, this can be expressed as a matrix multiplication. Activation functions, applied after the weighted sum, introduce non-linearity to the network.

Training a neural network involves adjusting weights to minimize the difference between predicted and actual outputs. This process, often achieved through backpropagation and optimization algorithms, again heavily relies on linear algebra concepts to efficiently compute gradients and update weights throughout the network.

Weightmatrix * input matrix + bias matrix = output matrix

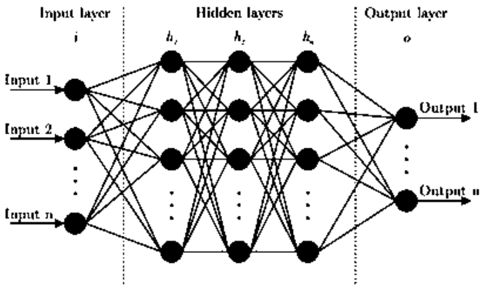


Fig 2. Neural Network Representation

*Source*:https://i0.wp.com/neptune.ai/wp-content/uploads/2022/10/Basic-neural-network.png

**5.2 Image Processing**: Image processing involves manipulating and analyzing images to enhance or extract information. Images are composed of pixels, and each pixel can be considered a data point. In grayscale images, pixel values are often represented as a matrix, where each element corresponds to the intensity at a specific position (Oberholzeret al., 1996).

In color images, colors are typically represented using the RGB (Red, Green, Blue) model. Each color channel can be considered as a matrix, and linear algebra operations are applied independently to each channel. Convolution operations, fundamental in image processing, can be efficiently implemented using linear algebra. Convolution involves sliding a filter (kernel) over the image and computing dot products at each position, which can be expressed as a matrix multiplication. Filtering operations, such as blurring or sharpening, are often performed using convolution. Linear algebra facilitates the application of filters to images, influencing pixel values based on neighboring elements (Hlavac,2011). Linear transformations, represented by matrices, are used for tasks like rotation, scaling, and skewing. These transformations can be applied to images, altering their appearance.

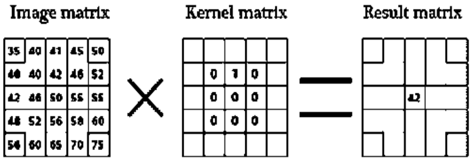


Fig 3 Matrix convolution Process

*Source*:https://www.researchgate.net/publication/343419411/figure/fig3/AS:920790337867782@1596544932844/Image-Matrix-Convolution-Process-with-a-Kernel-Matrix.png

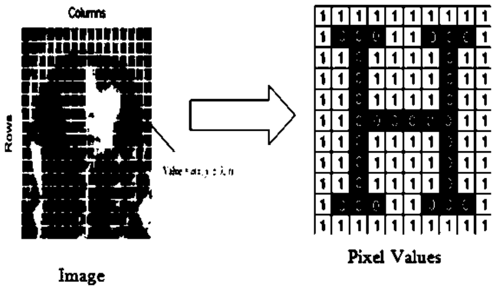
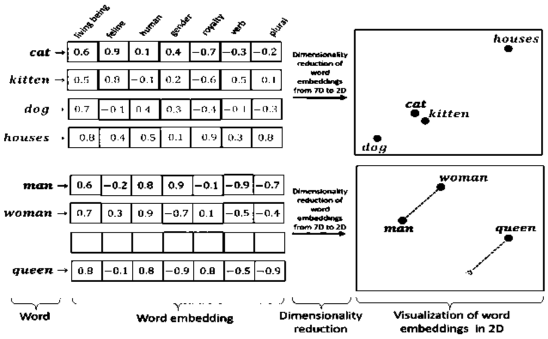


Fig 4 Image processing

*Source*:https://www.researchgate.net/publication/283800375/figure/fig1/AS:903419875319808@1592403491850/Image-pixel-generation-In-Digital-Image-Processing-the-digital-image-feature-extraction.ppm

**5.3 Word Embedding:** Word embedding is a technique used in natural language processing (NLP) to represent words as vectors in a continuous vector space (Lebret, 2016). The core idea is to map words to high-dimensional vectors in a way that captures semantic relationships between words. Linear algebra plays a central role in the mathematics behind word embeddings. Each word is represented as a vector, often with hundreds of dimensions. This vector captures the word's semantic information.For example, the word "king" might be represented as a vector, and the vector arithmetic "king - man + woman" can be expected to be close to the vector representation of "queen."

Similarity between word vectors is often measured using cosine similarity. It's a linear algebraic measure that calculates the cosine of the angle between two vectors. Higher cosine similarity indicates greater similarity in meaning.



| | living being | feline | human | gender | royalty | verb | plural |
|---|---|---|---|---|---|---|---|
| cat → | 0.6 | 0.9 | 0.1 | 0.4 | −0.7 | −0.3 | −0.2 |
| kitten → | 0.5 | 0.8 | −0.1 | 0.2 | −0.6 | 0.5 | 0.1 |
| dog → | 0.7 | −0.1 | 0.4 | 0.3 | −0.4 | −0.1 | −0.3 |
| houses → | 0.8 | 0.4 | 0.5 | 0.1 | 0.9 | 0.3 | 0.8 |
| man → | 0.6 | −0.2 | 0.8 | 0.9 | −0.1 | −0.9 | −0.7 |
| woman → | 0.7 | 0.3 | 0.9 | −0.7 | 0.1 | −0.5 | −0.4 |
| | | | | | | | |
| queen → | 0.8 | −0.1 | 0.8 | −0.9 | 0.8 | −0.5 | −0.9 |

Fig. 5 Word Embedding

*Source*: https://storage.googleapis.com/coderzcolumn/static/tutorials/artificial_intelligence/word_embeddings.jpg

**Singular Value Decomposition (SVD):** Singular Value Decomposition (SVD) in linear algebra is a factorization method that decomposes a matrix A into three matrices U, D and V’ as A = UDV’. U contains the left singular vectors, V’ contains the right singular vectors, and D is a diagonal matrix with singular values. Retaining only a subset of singular values and corresponding vectors allows for a lower-rank approximation of the original matrix. (Ashraf, 2022)

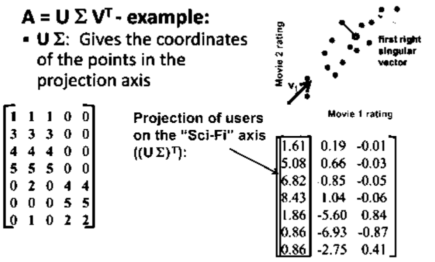


Fig 6 Represent SVD method.

*Source*:https://i.stack.imgur.com/hyQ8Q.png

**5.6 Principal value Decomposition:**

Principal Component Analysis (PCA) is a dimensionality reduction technique widely used in data analysis and machine learning (Hasan etal., 2021). In the context of linear algebra, PCA involves transforming the original data into a new set of orthogonal axes called principal components. PCA is performed using linear algebra as:

Given a dataset withnfeatures, the first step is to compute the covariance matrix, which captures the relationships between different features. The covariance matrix C = [Cij] is a symmetric matrix where each elementrepresents the covariance between feature ith and feature jth. The next step is to find the eigenvalues and eigenvectors of the covariance matrix. The eigenvectors represent the directions of maximum variance in the data, and the eigenvalues indicate the magnitude of variance along these directions. The covariance matrixC can be decomposed as C = Q D Q’, where Q is a matrix of eigenvectors, and D is a diagonal matrix with eigenvalues. Arrange the eigenvectors in Q based on the corresponding eigenvalues in D in descending order. The eigenvectors corresponding to the highest eigenvalues capture the most variance in the data. Select the top k eigenvectors to form the matrix W, where k is the desired dimensionality of the reduced dataset.Thus, PCA leverages linear algebra concepts such as covariance matrices, eigenvalue decomposition, and matrix multiplication to identify the principal components that capture the most significant variations in the data (Erichson,2016). By selecting a subset of these components, PCA reduces the dimensionality of the dataset while retaining the essential information. This can be particularly useful for visualization, noise reduction, and improving the efficiency of machine learning algorithms.

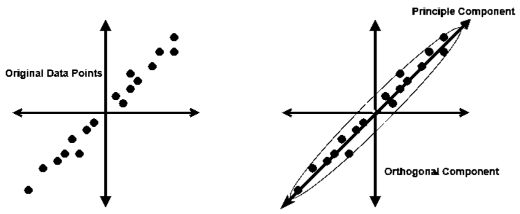


Fig. PCA Representation

*Source*:https://www.baeldung.com/wpcontent/uploads/sites/4/2020/06/PrincipleComponent.svg

**5.7 Recommended System**: A recommendation system in the context of linear algebra typically involves collaborative filtering or content-based filtering. Collaborative filtering represents users and items as vectors in a high-dimensional space, using techniques like matrix factorization for model representation. Content-based filtering relies on feature vectors for items and users, with similarity measures calculated through linear algebra operations. Hybrid models may combine these approaches. Linear algebra plays a crucial role in efficiently representing users, items, and their interactions, enabling accurate and scalable recommendation algorithms (Martinssonetal., (2020).

5.8 **Linear Regression:** Linear regression is a statistical method used for modeling the relationship between a dependent variable and one or more independent variables. In the context of linear algebra, linear regression can be expressed and understood through matrix notation (Meyer, C. D. 2023). linear regression is formulated using linear algebra as:

Model

$y = b0 + b1\, x + E$

y is the dependent variable,xis the independent variable,b0 is the y-intercept,b1 is the slope and E represents the error term.

In Matrix Form $Y = X\, b + E$

Where Yis a column vector representing the observed values of the dependent variable,X is a matrix where the first column is a column of ones (for the intercept) and the second column contains the values of the independent variable, beta is a column vector containing the parameters b0 and b1 and E is a column vector representing the error term. Similarly for Multiple Linear Regression hold.

This formula involves matrix multiplication and inversion, demonstrating the role of linear algebra in solving the linear regression problem.

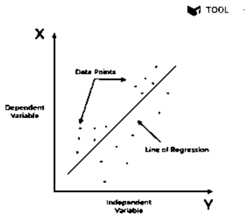


Fig 7: Representation of Regression

*Source*: https://images.spiceworks.com/wp-content/uploads/2022/04/07040339/25-4.png

## 6. FUTURE PROSPECTUS OF LINEAR ALGEBRA IN AI

Predicting the future of artificial intelligence (AI) in the context of linear algebra involves considering how advancements in this field may shape the development and applications of AI technologies (Zhang, 2021). Deep learning, a subfield of AI heavily reliant on linear algebra, is likely to see continued advancements. Innovations in neural network architectures, optimization algorithms, and parallel computing could enhance the capabilities of deep learning models.

(i) Efficient Model Training:Efforts to make deep learning models more efficient in terms of computation and memory requirements are expected. Techniques like neural architecture search and model compression, which involve linear algebra concepts, may become more prevalent.

(ii) Interdisciplinary Applications: The intersection of linear algebra and other disciplines, such as physics, biology, and materials science, may lead to novel AI applications. Cross-disciplinary research could uncover new ways of applying linear algebra techniques for solving complex problems.

(iii) Explainability and Interpretability: Addressing the interpretability of AI models is crucial for wider adoption. Future research might focus on incorporating linear algebra-based methods to make AI models more transparent and interpretable, helping to build trust in AI systems.

(iv) Quantum Computing Impact:As quantum computing advances, linear algebra will play a vital role in quantum machine learning. Quantum algorithms for linear algebra tasks may significantly accelerate certain AI computations, leading to breakthroughs in AI capabilities.

While predicting the future of AI is challenging, the ongoing importance of linear algebra in advancing AI technologies is evident. Continued research, interdisciplinary collaboration, and ethical considerations will likely shape the trajectory of AI development in the coming years.

## 7. CONCLUSION

The integration of linear algebra principles in AI has revolutionized the way we approach problem-solving and data manipulation. By utilizing matrices, vectors, and linear models, AI algorithms can unravel layers of complexities and extract meaningful insights. Linear algebra provides the backbone for various AI applications, enabling automated image recognition, voice analysis, and data processing. Ultimately, the seamless fusion of linear algebra and AI leads us into a future where machines possess the capacity for critical thinking and intelligent decision-making.

## 8. REFERENCES

1. Aggarwal, C. C., Aggarwal, L. F., &Lagerstrom-Fife. (2020). Linear algebra and optimization for machine learning (Vol. 156). Springer International Publishing.
2. Ashraf, M., De Filippis, V., & Aslam Siddeeque, M. (2022). Applications of Linear Algebra to Numerical Methods. In Advanced Linear Algebra with Applications (pp. 333-364). Singapore: Springer Nature Singapore.
3. Bellet, A., Habrard, A., &Sebban, M. (2013). A survey on metric learning for feature vectors and structured data. *arXiv preprint arXiv:1306.6709*.
4. Cristianini, N., &Scholkopf, B. (2002). Support vector machines and kernel methods: the new generation of learning machines. *Ai Magazine*, *23*(3), 31-31.
5. Dhanalakshmi, P. (2021). Linear algebra for machine learning. In Artificial intelligence theory, models, and applications (pp. 405-428). Auerbach Publications.
6. Dhillon, P. S., Foster, D. P., & Ungar, L. H. (2015). Eigenwords: spectral word embeddings. *J. Mach. Learn. Res.*, *16*, 3035-3078.
7. Erichson, N. B., Voronin, S., Brunton, S. L., &Kutz, J. N. (2016). Randomized matrix decompositions using R. arXiv preprint arXiv:1608.02148.
8. Eves, H. W. (1980). Elementary matrix theory. Courier Corporation.
9. Gentle, J. E. (2012). Numerical linear algebra for applications in statistics. Springer Science & Business Media.
10. Golub, G. H., & Van der Vorst, H. A. (2000). Eigenvalue computation in the 20th century. Journal of Computational and Applied Mathematics, 123(1-2), 35-65.
11. Golub, G. H., & Van Loan, C. F. (2013). Matrix computations. JHU press.
12. Goodfellow, I., Bengio, Y., & Courville, A. (2016). Deep learning. MIT press.

13. Hasan, B. M. S., &Abdulazeez, A. M. (2021). A review of principal component analysis algorithm for dimensionality reduction. *Journal of Soft Computing and Data Mining*, *2*(1), 20-30.
14. Hlavac, V. (2011). Fundamentals of Image Processing. Optical and Digital Image Processing: Fundamentals and Applications.
15. Lax, P. D. (2007). Linear algebra and its applications (Vol. 78). John Wiley & Sons.
16. Lebret, R. P. (2016). Word embeddings for natural language processing (No. THESIS). EPFL.
17. Ma, Y., Wang, Z., Yang, H., & Yang, L. (2020). Artificial intelligence applications in the development of autonomous vehicles: A survey. IEEE/CAA Journal of AutomaticaSinica, 7(2), 315-329.
18. Martinsson, P. G., &Tropp, J. A. (2020). Randomized numerical linear algebra: Foundations and algorithms. Acta Numerica, 29, 403-572.
19. Mehrotra, K., Mohan, C. K., &Ranka, S. (1997). Elements of artificial neural networks. MIT press.Elements of artificial neural networks. MIT press.
20. Meyer, C. D. (2023). Matrix analysis and applied linear algebra (Vol. 188). Siam.
21. Meyer, C. D. (2023). Matrix analysis and applied linear algebra (Vol. 188). Siam.
22. Najafabadi, M. M., Villanustre, F., Khoshgoftaar, T. M., Seliya, N., Wald, R., &Muharemagic, E. (2015). Deep learning applications and challenges in big data analytics. *Journal of big data*, *2*(1), 1-21.
23. Nascimento, A. M., Vismari, L. F., Molina, C. B. S. T., Cugnasca, P. S., Camargo, J. B., de Almeida, J. R., ... &Hata, A. Y. (2019). A systematic literature review about the impact of artificial intelligence on autonomous vehicle safety. IEEE Transactions on Intelligent Transportation Systems, 21(12), 4928-4946.
24. Oberholzer, M., Östreicher, M., Christen, H., &Brühlmann, M. (1996). Methods in quantitative image analysis. Histochemistry and cell biology, 105, 333-355.M., Östreicher, M., Christen, H., &Brühlmann, M. (1996). Methods in quantitative image analysis. Histochemistry and cell biology, 105, 333-355.
25. Olver, P. J., Shakiban, C., &Shakiban, C. (2006). Applied linear algebra (Vol. 1). Upper Saddle River, NJ: Prentice Hall.
26. Pannu, A. (2015). Artificial intelligence and its application in

different areas. Artificial Intelligence, 4(10), 79-84.

27. Spector, P. (2008). *Data manipulation with R*. Springer Science & Business Media.
28. Srebro, N. (2004). Learning with matrix factorizations.
29. Stoll, R. R. (2013). Linear algebra and matrix theory. Courier Corporation.
30. Strang, G., & Algebra, L. (1980). its Applications. Academic Press, New York, 14, 181208.
31. Suthaharan, S. (2016). Machine learning models and algorithms for big data classification. *Integr. Ser. Inf. Syst*, *36*, 1-12.
32. Velho, L., Frery, A. C., & Gomes, J. (2009). Image processing for computer graphics and vision. Springer Science & Business Media.
33. Zhang, C., & Lu, Y. (2021). Study on artificial intelligence: The state of the art and future prospects. Journal of Industrial Information Integration, 23, 100224
34. Zhang, Q., Xin, C., & Wu, H. (2021). GALA: Greedy computation for linear algebra in privacy-preserved neural networks. arXiv preprint arXiv:2105.01827.
35. ***Images***:
36. https://www.google.com/url?sa=i&url=https%3A%2F%2Fen.wikipedia.org%2Fwiki%2FEigenvalues_and_eigenvectors&psig=AOvVaw3vsLdziHp1Fv6oNK0iQZlR&ust=1706957108622000&source=images&cd=vfe&opi=89978449&ved=0CBMQjhxqFwoTCODO7fa8jIQDFQAAAAAdAAAAABAI
37. https://www.researchgate.net/publication/283800375/figure/fig1/AS:903419875319808@1592403491850/Image-pixel-generation-In-Digital-Image-Processing-the-digital-image-feature-extraction.ppm
38. https://i0.wp.com/neptune.ai/wp-content/uploads/2022/10/Basic-neural-network.png?resize=651%2C368&ssl=1
39. https://www.researchgate.net/publication/343419411/figure/fig3/AS:920790337867782@1596544932844/Image-Matrix-Convolution-Process-with-a-Kernel-Matrix.png
40. https://storage.googleapis.com/coderzcolumn/static/tutorials/artificial_intelligence/word_embeddings.jpg
41. https://www.baeldung.com/wp-content/uploads/sites/4/2020/06/PrincipleComponent.svg
42. https://images.spiceworks.com/wp-content/uploads/2022/04/07040339/25-4.png
43. https://i.stack.imgur.com/hyQ8Q.pngtps