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Optimization of Production Inventory under Weibull Deterioration and Price Discount Conditions: A Mathematical Modeling and Sensitivity Analysis Approach

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Abstract

This study presents an integrated production-inventory model that incorporates Weibull deterioration and price discount considerations to optimize inventory strategies for perishable goods. The model is developed under the assumptions of deterministic demand, continuous replenishment, and the absence of shortages, while deterioration is governed by a three-parameter Weibull distribution. A mathematical formulation based on differential equations is established to represent inventory behavior during both production and non-production periods. The model accounts for setup costs, holding costs, production costs, deterioration losses, and discount incentives to derive a Total Variable Cost (TVC) function. Analytical methods are employed to determine the optimal cycle time that minimizes overall cost. A numerical illustration using realistic parameter values demonstrates the efficiency and applicability of the proposed model. In addition, a comprehensive sensitivity analysis evaluates the impact of key parameters such as setup cost, production rate, demand level, and deterioration rate on optimal inventory policies. The findings highlight the practical relevance of the proposed framework for decision-making in industries managing deteriorating products. The results provide valuable managerial insights to improve cost efficiency, enhance production planning, and promote operational sustainability.

Keywords: Inventory management, Weibull deterioration, price discount, optimization model, production planning, sensitivity analysis, perishable goods, total variable cost, inventory dynamics, holding cost.

Introduction

The Production Inventory Model with Weibull Deterioration and Price Discount is a mathematical framework used in inventory management to optimize production and inventory decisions for items subject to deterioration, while also considering price discounts. This model is particularly relevant for industries dealing with perishable goods or items prone to deterioration over time, such as food products or certain types of chemicals. In this model, the key components include demand, deterioration, production, inventory holding and ordering costs, as well as price discounts. The demand for the product is typically assumed to follow a known pattern and deterioration is often modeled using the Weibull distribution, which captures the probability distribution of the time until an item deteriorates. The primary objective of the Production Inventory Model is to determine the optimal production quantity and order quantity that minimize total inventory costs

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while meeting demand and accounting for deterioration and price discounts. This involves finding the balance between production costs, inventory holding costs and ordering costs, while also considering the impact of deteriorating inventory on customer satisfaction and potential revenue losses.

Price discounts introduce an additional layer of complexity into the production-inventory model. These discounts are typically offered either for purchasing in larger quantities or for placing orders during specific periods. Integrating price discounts into the decision-making process requires evaluating the trade-offs between the savings achieved through discounts and the potential increase in costs associated with holding excess inventory or placing more frequent orders. To solve the Production Inventory Model with Weibull Deterioration and Price Discount, various optimization and computational techniques can be employed, including dynamic programming, numerical analysis, and heuristic algorithms. These methods aim to identify the optimal production and ordering policies that maximize profitability while minimizing total costs over a defined planning horizon.

By applying this model, organizations can make data-driven decisions regarding production scheduling, inventory replenishment, and pricing strategies. This allows businesses to manage inventory more efficiently, reduce losses due to deterioration, and capitalize on price discount opportunities to enhance profitability and competitive advantage. Overall, the Production Inventory Model with Weibull Deterioration and Price Discount represents a valuable analytical tool for industries where effective inventory management is critical to operational performance and long-term business success.

ASSUMPTIONS AND NOTATIONS”

The following assumptions are considered in the development of the Production Inventory Model with Weibull Deterioration and Price Discount:

Assumptions:

(i) The demand for the product is known, constant, and does not vary with time.

(ii) Shortages or stock-outs are not permitted.

(iii) The planning horizon is assumed to be infinite.

(iv) Each unit of the product, once produced, is immediately available to meet demand.

(v) Items that are not in perfect condition may be sold at a discounted price.

(vi) Repair or replacement of deteriorated items is not allowed.

Notations:

p:Production rate per unit time

d: Actual demand rate of the product per unit time, where $d < p$

A: Setup cost per production run

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Deterioration rate (unit/unit time), $\theta = \alpha\beta(t-\gamma)^{\beta-1}$ where $0<\alpha<1$, $\beta>1, 0<\gamma<1$ where an is the scale parameter, p is the shape parameter and y is the location parameter of the Waybill three parameter deterioration.

H: "Constant inventory carrying cost per unit per unit time.

k Production cost per unit.

l Price discount per unit cost.

T Optimal cycle time.

T1: Production period.

T2 Time during which there is no production, i.e., T2=T-T1."

Ix(t): Inventory level for product during the production period, i.e. $0\leq t\leq T1$

I2{t): Inventory level of the product during the period when there is no production, i.e. $T1\leq t\leq T$

I(M): Maximum inventory level ofthe product.

TVC{T): Total cost/unit time.

MATHEMATICAL MODEL

At time t = 0, there are no items in stock. Production and supply both begin at the same time and production stops when the highest stock level, 1(M), is reached at time t=Tx. During this time, the inventory grew at a rate of p/d and it didn't break down. After time 7J, the units that have been delivered start to break down and the supply goes on at the markdown rate. As long as interest in the item stays the same, the number of items in stock will go down until there are none left, at which point the production run will begin. So, the accompanying different conditions can be used to deal with the item's stock level at time t over the range [0,T].

$dI_1(t)/dt = p - d \quad 0\leq t\leq T1$……70

And

$dI_2(t)/dt + \theta I_2(t) = -d$…..$T1\leq t\leq T$

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Where $\theta = \alpha\beta(t-\gamma)^{\beta-1}$ $0<\alpha<1$, $\beta>1, 0<\gamma<1$

The scale parameter is x and the shape parameter is y. The location parameter is y.

Here the boundary conditions are $I1(0)= I_2(T_2) = 0$

Using the boundary condition I1(0) = 0 solution of equation (1) is

$I1(t) = (p - d)\, t$…….$0\leq t\leq T1$

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Equation (71) is a linear differential equation.

Integrating Factor of equation (71) is

$e\int e^{\alpha\beta(t-\gamma)^{\beta-1}} == e^{\alpha(t-\gamma)^{\beta}}$

using the integration factor from above, the answer to equation (71) is

$I_2(t)e^{\alpha(t-\gamma)^{\beta}} = \int -d \; e^{\alpha(t-\gamma)^{\beta}} + c$

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Since 0 a 1, we can ignore the terms in the exponential function's expansion that have to do

$$\Rightarrow I_2(t)e^{\alpha(t-\gamma)^\beta} = -d\int \{1+\alpha(t-\gamma)^\beta\}dt + c$$

$$\Rightarrow I_2(t)e^{\alpha(t-\gamma)^\beta} = -d\left\{t+\frac{\alpha(t-\gamma)^{\beta+1}}{\beta+1}\right\} + c$$

with the second and higher powers of a. This gives us T.,

Now, going back to the first condition I2 (T2) = 0above, we can find the answer we need for equation (71) as

$I_2(t)e^{\alpha(t-\gamma)^\beta}[\mathrm{T2} - \mathrm{t} + \frac{\alpha}{\beta+1}\{(T_2-\gamma)^{\beta+1} - \ t-\gamma)^{\beta+1}]$

Since 0 < a < 1, If we ignore the parts of the exponential function that have to do with the second and higher powers of a, we get,

$$\Rightarrow I_2(t) = d\{1-\alpha(t-\gamma)^\beta\}\left[T_2 - t + \frac{\alpha}{\beta+1}\{(T_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}\}\right]$$

$$= d\left[T_2 - t + \frac{\alpha}{\beta+1}\{(T_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}\} - T_2\alpha(t-\gamma)^\beta + t\alpha(t-\gamma)^\beta \right.$$
$$\left. - \frac{\alpha^2}{\beta+1}\{(t-\gamma)^\beta(T_2-\gamma)^{\beta+1} - (t-\gamma)^{2\beta+1}\}\right]$$

Since 0 < a < 1, Leaving out the parts about the second and higher powers of an in the above, we get,

$$I_2(t) = d\left[T_2 - t + \frac{\alpha}{\beta+1}(T_2-\gamma)^{\beta+1} - \alpha T_2(t-\gamma)^\beta \right.$$
$$\left. + \alpha t(t-\gamma)^\beta - \frac{\alpha}{\beta+1}(t-\gamma)^{\beta+1}\right], T_1 \le t \le T$$

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The set up cost per unit time is

SC= A/T…………74

The Holding Cost is

$$HC = \frac{1}{T}\left[\int_0^{T_1} h(t)I_1(t)dt + \int_0^{T_2} h(t)I_2(t)dt\right]$$

$$\Rightarrow HC = \frac{1}{T}\left[h\int_0^{T_1}(p-d)tdt\right] + \frac{hd}{T}\int_0^{T_2}\left[T_2 - t + \frac{\alpha}{\beta+1}(T_2-\gamma)^{\beta+1} \right.$$
$$\left. - \frac{\alpha}{\beta+1}(t-\gamma)^{\beta+1} - \alpha T_2(t-\gamma)^\beta + \alpha t(t-\gamma)^\beta\right]dt$$

Integrating the above we get

$$HC = \frac{h(p-d)T_1^2}{2T} + \frac{hd}{T}\left[\frac{T_2^2}{2} - \frac{2\alpha(T_2-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T_2(T_2-\gamma)^{\beta+1}}{(\beta+1)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T_2(-\gamma)^{\beta+1}}{(\beta+1)}\right] \quad \text{........75}$$

Let's say what T1 and T2 mean in terms of T.

$I_1(T_1) = I_2(0)$

= (p – d) T1 = d [T2 $+\frac{\alpha}{\beta+1}(T_2-\gamma)^{\beta+1} - \frac{\alpha}{\beta+1}(-\gamma)^{\beta+1} - \alpha T_2(-\gamma)^{\beta} + \alpha t(-\gamma)^{2\beta+1}\Big]$

Since 0 a 1, we can ignore the terms in the above equation with a to get a good answer.

(p- d) T1 = dT2

= T – T2/T2 = d / p-d

= T/T2 =p/p-d

= T2 = (p- d) T /p = xT , wherelet x = p – d /p

= T1 = T – T2 =dT /p

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By plugging these numbers into equation (76), we get

$$HC = \frac{hdxT}{2} - \frac{2\alpha hd(xT-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd\alpha x(xT-\gamma)^{\beta+1}}{(\beta+1)} + \frac{2\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd\alpha x(-\gamma)^{\beta+1}}{(\beta+1)} \quad \text{………….77}$$

The cost of making something per unit of time is

PC =pkT1/T = pk dT /Tp = kd

……….78

Deterioration cost:

The difference between the most units in stock and the number of units used to meet demand is the number of units that break during a cycle. So, the cost of breaking down per unit of time isgiven as

DC=k/T [I_2(0) - $\int_0^{T_2}$ d dt

$= \mathrm{kd}/\mathrm{T}[\frac{\alpha}{\beta+1}(T_2-\gamma)^{\beta+1} - \frac{\alpha}{\beta+1}(-\gamma)^{\beta+1} - \alpha T_2(-\gamma)^{\beta}\Big]$

When we use the values of T1 and T2 in terms of T, we get

DC= $\mathrm{kd}/\mathrm{T}[\frac{\alpha}{\beta+1}(\mathrm{x}T-\gamma)^{\beta+1} - \frac{\alpha}{\beta+1}(-\gamma)^{\beta+1} - \alpha xT(-\gamma)^{\beta}\Big]$

………….79

Price discount:

Price cuts are given as a percentage of the cost to make the units in the Period (0,T2)

= PD= Kl/T$\int_0^{T_2}$ d dt

= kldT2 /T = kldxT/ T = kl dx

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So, to figure out the average total cost per unit of time,

$$TVC(T) = PC + SC + HC + PD + DC$$

$$= kd + \frac{A}{T} + \frac{hdxT}{2} - \frac{2\alpha hd(xT-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd\alpha x(xT-\gamma)^{\beta+1}}{(\beta+1)} + \frac{2\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd\alpha x(-\gamma)^{\beta+1}}{(\beta+1)} + \frac{kd}{T}\left[\frac{\alpha(xT-\gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha(-\gamma)^{\beta+1}}{(\beta+1)} - \alpha xT(-\gamma)^{\beta}\right] + kldx$$

$$\Rightarrow TVC(T) = kd + \frac{A}{T} + \frac{hdxT}{2} - \frac{2\alpha hd}{(\beta+1)(\beta+2)T}\left\{(xT-\gamma)^{\beta+2} - (-\gamma)^{\beta+2}\right\} + \frac{hd\alpha x}{(\beta+1)}\left\{(xT-\gamma)^{\beta+1} + (-\gamma)^{\beta+1}\right\} + \frac{kd\alpha}{(\beta+1)T}\left\{(xT-\gamma)^{\beta+1} - (-\gamma)^{\beta+1}\right\} - \alpha kdx(-\gamma)^{\beta} + kldx$$

To find the lowest total cost, we figure out how much T is from

$$\frac{d}{dT}(TVC(T)) = 0$$

$$\Rightarrow \frac{-A}{T^2} + \frac{hdx}{2} - \frac{2\alpha hdx(xT-\gamma)^{\beta+1}}{(\beta+1)T} + \frac{2\alpha hd(xT-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^2} - \frac{2\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^2} + \alpha hdx^2(xT-\gamma)^{\beta} + \frac{\alpha kdx(xT-\gamma)^{\beta}}{T}$$

$\frac{\alpha kd(xT-\gamma)^{\beta+1}}{(\beta+1)T^2} + \frac{\alpha kd(-\gamma)^{\beta+1}}{(\beta+1)T^2} = 0$**…………81**

Using the calculated value of T from (14) will make the TVC as small as possible.

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$$\frac{d^2}{dT^2}(TVC(T)) > 0$$

$$\Rightarrow \frac{2A}{T^3} - \frac{2\alpha hdx}{(\beta+1)T^2}\left[Tx(\beta+1)(xT-\gamma)^{\beta} - (xT-\gamma)^{\beta+1}\right]$$

$$+\frac{2\alpha hd}{(\beta+1)(\beta+2)T^4}\left[T^2x(\beta+2)(xT-\gamma)^{\beta+1} - 2(xT-\gamma)^{\beta+2}T\right] + \frac{4\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^3}$$

$$+\alpha hdx^3\beta(xT-\gamma)^{\beta-1} + \frac{\alpha kdx}{T^2}\left[Tx\beta(xT-\gamma)^{\beta-1} - (xT-\gamma)^{\beta}\right]$$

$$-\frac{\alpha kd}{(\beta+1)T^4}\left[T^2x(\beta+1)(xT-\gamma)^{\beta} - 2T(xT-\gamma)^{\beta+1}\right] - \frac{2\alpha kd(-\gamma)^{\beta+1}}{(\beta+1)T^3} > 0$$

………82NUMERICAL EXAMPLE:

Let ^ = Rs 2000/set up, p = 200 units/unit time, d = 50 unit/unit time,or = 0.6,= 10,y =0.4,A: =/ls60/unit, l = 0.05,h = 2. The conditions in (9), (13), (14) and (15) are very hard to understand. You can solve them by writing code in Mathematica-5.1. Using these qualities for condition 14, we

getT* = 1.83327. Using this value of T*in equation (15) we get d^2/dt^2 (TVC(T))= 4638 >0 which is positive. So, this value of T* will make the total average variable cost as low as possible. Again, equations 9 and 13 tell us that the best values for HC* are 71.4766 and TVC* is 4342.31.

SENSITIVITY ANALYSIS

A sensitivity analysis has been done by changing one system parameter at a time while leaving the others the same. The above example was used to get the original values of all the parameters for sensitivity analysis.

Table-1 for sensitivity analysis

Parameter	% change	T*	HC*	TVC*
	-50	1.74542	66.7206	3784.86
	-40	I.76837	67.8705	3898.68
	-30	1.7878.4	67.898	4011.14
	-20	1.80478	69.8197	4122.47
	-10	1.81978	70.6758	4232.82
	0	1.83327	11.476·	4342.31
	10	l.845533	72.2321	4451.04
	20	1.85677	72.95	4559.07
	30	1.86715	73.6362	4666.48
A	40	1.87681	74.2963	4773.32

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	50	1.88584	74.9333	4879.13
	-50	2.74378	71.2062	4059.29
	-40	2.35407	7l.3571	4160.37
	-30	2.13724	71.4766	4236.18
	-20	1.99913	71.5737	4295.14
	-10	1.90346	71.6548	4342.31
	0	1.83327	71.7226	4380.9
	10	1.77958	71.7808	44l3.06
	20	1.73719	71.8308	4440.28
	30	1.70286	70.35.21	4463.6
	40	1.6745	70.7416	4483.82
P	50	1..65067	70 .0096	3192.19
	-50	1.64963	65.1875	3483.95
	-40	1.671676	71.4766	3772.58
	-30	l.70909	77.7166	4058.58
	-20	1.74608	83.9111	4342.31
	-10	1.78748	90.0611	4624..02
	0	1.83327	96.1697	4903.91
	10	1.8835'5	102.236	4518.2l
	20	l.9J854	70.35.21	4545.768
	30	l.99853	70.7416	3917.76
	40	2.06395	70 .0096	4483.82
d	50	2.13528	70 .0096	3192.19
	-50	1.91922	73.494	4430.77
	-40	1.8961	72.4787	4388.56
	-30	l.87684	72.1252	4378.27
	-20	l.8·6037	71.855	4361.53
	-10	1.846	71.6436	4351.75
	0	1.83327	71.4766	4342.31
	10	1.8.2186	71.3411	4335.05
	20	1.81153	71.228	4328.68
	30	1.80209	78.3901	438856
α	40	1.791342	71.6436	4378.27

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	50	1.7854	71.4766	4361.53
	-50	1.8146	74.7235	4294.47
	-40	1.82191	73.8507	4306.9
	-30	1.8251.2	73.1233	4317.5
	-20	1.821l6	72.56	4326.74
	-10	1.830816	71.9579	4334.94
	0	l.83327	71.476	4342.11
	10	1.83541	71.0451	4349.01
	20	1.83731	70.6544	4355.15
	30	1.8146	70.2971	4360.81
β	40	1.82191	69.9691	4366.08
	50	1.8251.2	69.6656	4370.99
	-50	1.83327	62..8562	4527.97
	-40	t.88229	64.5606	4486.CU
	-30	1.93141	66.27561	4446.69
	-20	1.98062	68.0004	4409..78
	-10	2.02991	69.7341	4375.06
	0	2.0793	71.4766	4342.031
	10	1.82191	73.2268.	4311.31
	20	1.8251.2	74.9846	4428.77
	30	1.821l6	76.7485	4253.27
	40	1.830816	78.5169·	4225.2
γ	50	1.98062	80.289	4196.7
	-50	1.91385	77.0455	2740.91
	-40	1.89259	75.4228	3063.71
	-30	1.87462	74.1448	3138.92
	-20	1.859091	73.1014	3704.92
	-10	1.84544	72.2264	4023.99
	0	l.83327	71.4766	434.2.31
	10	1.8223l	70.8237	4660.01
	20	1.81234	70.2469	4977.19
	30	1.8032	69.7314	5293.93
k	40	l.79478	69.2672	5610.3
	50	l.78696	68.8446	5926.33

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The data presented represent a sensitivity analysis of key parameters within an inventory management model. Each entry corresponds to a percentage variation in a specific parameter, along with the resulting changes in the optimal order quantity, holding cost, and total variable cost. Parameter A demonstrates a consistent trend: as its value decreases, the optimal order quantity also declines, whereas holding costs and total variable costs rise. Conversely, an increase in parameter A leads to a higher optimal order quantity and a corresponding reduction in holding and total variable costs. A similar relationship is observed for parameter P, where decreasing values are associated with reduced optimal order quantities and elevated holding and total variable costs, while increasing values have the opposite effect—enhancing order quantities and reducing overall costs. Subsequent sensitivity analyses conducted for parameters d, α, β, γ, and k reveal a comparable pattern. Decreases in these parameter values generally result in lower optimal order quantities and higher cost components, whereas increases lead to greater order quantities accompanied by cost reductions. Overall, this sensitivity analysis provides meaningful insights into how variations in model parameters influence inventory performance and cost efficiency. By understanding these interdependencies, businesses can fine-tune their production and inventory control strategies to optimize operational performance, cost-effectiveness, and profitability in dynamic and competitive market conditions.

Conclusion

The present study develops and analyzes a comprehensive Production Inventory Model that integrates Weibull deterioration and price discount mechanisms, offering an effective decision-support tool for managing perishable inventories. By incorporating deterioration effects through a flexible three-parameter Weibull distribution, the model realistically represents time-dependent declines in product quality. The inclusion of price discounting reflects practical business conditions, providing a realistic basis for optimizing order quantities and minimizing total costs. The mathematical framework, established through differential equations and boundary conditions, determines optimal production cycle time, inventory levels, and total variable costs. The numerical illustration validates the model's capability to minimize overall costs by identifying the optimal cycle time. Sensitivity analysis further enhances the study by revealing how variations in parameters such as production rate, demand rate, setup cost, deterioration rate, and discount rate affect optimal decisions. The results highlight the model's flexibility and robustness under dynamic operational environments. This model offers a valuable framework for industries dealing with deteriorating inventories, including food processing, pharmaceuticals, and chemical manufacturing. It provides a cost-effective strategy for balancing production, holding, and procurement decisions, ultimately improving operational efficiency, reducing wastage, and enhancing profitability. Future research can extend this model by incorporating stochastic demand, multi-item systems, partial backordering, and varying discount structures. Such extensions would further enhance the model's applicability and provide deeper insights into sustainable and adaptive inventory management practices in real-world business settings.

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