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An Optimized Algorithm for Flow Shop Scheduling with Rental Machines

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ABSTRACT:
Efficient scheduling is critical for optimizing resource allocation and minimizing operational costs in manufacturing and service industries. Flow shop scheduling problems (FSSPs) become more complex when machines are rented instead of owned, introducing additional constraints and cost considerations. This paper proposes an optimized approach to sequencing jobs on two rented machines to achieve minimum total rental cost while maintaining productivity. The study formulates the problem mathematically, incorporates setup times and probabilities and applies a modified Johnson's algorithm to determine the optimal job sequence. A computational analysis is performed across multiple problem sizes and compared with established heuristics, including Palmer (1985), Johnson (1954) and NEH (1983). Results demonstrate that the proposed algorithm consistently achieves lower rental costs than existing methods. This research provides a practical decision-support tool for industries facing capital constraints, helping reduce equipment rental expenses without compromising efficiency.

KEYWORDS: Flow shop scheduling; rental machines; job sequencing; Johnson's algorithm; optimization; manufacturing systems; heuristic comparison.

INTRODUCTION:

The process of scheduling is an essential and integral aspect of resource allocation, wherein the deployment of assets is carefully planned and executed to facilitate the execution of activities. The chief goal of scheduling is in order to identify the most optimal solution, taking into contemplation the pressing desire for optimum a specific purpose or outcome. The wellknown flow shop scheduling problem(FSSP) conforms evaluating the best sequence for

PRACTICAL SITUATION

The presence of various experimental and practical circumstances is commonly observed throughout everyday involvement in manufacturing and fabrication settings. These scenarios often require the execution of diverse tasks that involve the utilization of different types of industrial equipment. The weightage of jobs can be observed in various industries, including the cotton industry, leather manufacturing unit and textile factory. These industries serve as practical examples to understand the significance of different job roles and their contributions. Different varieties of cotton, shoes, jackets and fabric of varying sizes or qualities are carried out in diverse manufacturing facilities, reflecting the diverse range of consumer preferences and market

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demands. Due to a lack of finances in his early profession, one needs to rent the machines. For example, to start a pathology laboratory, much expensive equipment like a microscope, water bath, lab incubator, glucometer, blood cell counter, tissue diagnostics, etc., one does not buy these machines but instead take on rent. Renting enables saving capital investments, helping choose the right equipment for the job and access the latest technology.

Assumptions

- There is no room for any kind of transfer between two different machines, H_1 and H_2, because of processing of jobs which work autonomously in sequential H_1 H_2.
- Simultaneous processing of a single job by two machines is not feasible.
- Any alteration to the machines' path of action is strictly prohibited until the completion of said job becomes unattainable.
- Time spent for setting up and equipment break down are not factored into utilization calculations.

Rental Policy

The machines are rented on as needed basis and subsequently return them once they are no longer necessary. Specifically, the initial machine acquired through a rental agreement at the commencement of job processing. Subsequently, the second machine will be obtained on a rental basis once the initial job on the first machine has been completed.

PROBLEM FORMULATION

Consider the processing of jobs i (where i ranges from 1 to n) by two machines, denoted as H_1 and H_2. Take into account the processing time pertaining to probabilities P_{i1} &P_{i2} on the machines H_1 &H_2 denoted by h_{i1} and h_{i2}. Also, the setup times S_{i1} and S_{i2} pertaining to probabilities Q_{i1} &Q_{i2} on the machines H_1 and H_2 correspondingly. The model's mathematical representation can be expressed mathematically in the form of **Error! Reference source not found.**in a matrix-based format. In order to minimize capital expenditures for rented equipment, our mission is to pinpoint the optimum jobs $\{s_1\}$ sequence.

ALGORITHM

Step 1: Determine the processing times, named as H_{i1}& H_{i2}, for the machines H_1& H_2 respectively:

$$Hi1 = hi1 \times Pi1 - Si2 \times Qi2 \qquad (1)$$

$$Hi2 = hi2 \times Pi2 - Si1 \times Qi1 \qquad (2)$$

Step 2: While cutting down on the total amount of time elapsed, implement on Johnson's method (1954) to acquire the optimum string s_1.

Step 3: For computing the total elapsed time for string s_1, build a flow in-out table.

Step 4: Determine

$$l_2 = T_{i2} - \sum_{n=1}^{\infty} H_{i2} \quad (3)$$

Step 5: In order for machine H_2 to commence processing, the most recent time l_2 considered as the starting point for processing will be employed to generate a flow in-flow out table.
Step 7: Calculate utilization time $u_1(s_1)$ and $u_2(s_1)$ of machines H_1& H_2 by:

$$u_1(s_1) = \sum_{n=1}^{\infty} H_{i1} \quad (4)$$

$u_2(s_1) = T_{i2} - l_2$ □ (5)
Step 8: Finally, calculate
$r(s_1) = u_1(s_1) *c_1+u_2(s_1) *c_2$ (6)
NUMERICAL ILLUSTARTION
Taking into consideration, where processing durations separating to the setup times are specified in **Error! Reference source not found.**, assume five jobs and two machines. Four and six units of time are needed to hire machines H_1 and H_2, respectively. Our goal is to achieve optimal efficiency of sequencing jobs for execution on machines that may be rented for the most economical cost.
Solution:
In accordance with Step 1, TABLE III. presents an overview of the anticipated processing times on machines H_1 and H_2
COMPUTATIONAL EXPERIMENTS
In order to analyse the suggested heuristic approach, an arbitrary number of samples for multiple groups each of which has various number of jobs are taken. A total of eight groups, each consisting of job sizes 5, 10, 20, 30, 40, 50, 60 and 80 are created. Each group was then subjected to observation under five distinct tribulations, which were randomly generated. A comparison is made between the mean of overall rental cost in the proposed algorithm and the current make-span techniques of Palmer (1985),
Johnson (1954), NEH (1983) and Nailwal (*). The results are presented in TABLE VIII. and graph was plotted, as shown in Fig. 1, to illustrate the comparison. The findings indicate that, when compared to the remaining curves, the curve associated with the suggested approach has a lower trajectory. Notably, Palmer's algorithm demonstrates a significantly elevated curve compared to other existing approaches. Furthermore, the curve of NEH (1983) is closer than others to the proposed algorithm's curve. Moreover, To assess the quality of the suggested algorithm, calculation of error percentage for each problem follows a specific formula, denoted as E_{rr}. This formula is expressed as:
$[(R_\delta - R_\theta)/ R_\theta] \times 100$

In this case, R_δ represents the overall rental cost of all currently available algorithms, while R_θ represents overall rental cost associated with same job determined when utilizing the new algorithm and results are plotted in the graph below, which is depicted in Error! Reference source not found.

CONCLUSION:

This study demonstrates the importance of incorporating rental policies into classical flow shop scheduling problems. By integrating setup times, processing probabilities and cost structures, the proposed algorithm provides a more realistic approach to scheduling in resourceconstrained environments. The computational experiments show that our heuristic yields consistently lower rental costs compared to well-known scheduling techniques. Notably, the gap between the proposed method and NEH (1983) is smaller than with other techniques, indicating strong competitiveness. For practical applications, such as textile, cotton, leather and laboratory equipment rental industries, this approach can reduce costs and improve machine utilization. Future research can extend this work by including more machines, dynamic rental pricing and stochastic job arrivals, as well as applying metaheuristic techniques such as genetic algorithms or simulated annealing for further optimization.

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CONFLICT OF INTEREST

All authors confirm that they have no conflicts of interest.

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